Adaptation of *Physarum polycephalum* evolution for least-cost design of water distribution network

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ABSTRACT

This study presents a physarum polycephalum-inspired mathematical model for the solution of the problem of least cost design of water distribution systems. We propose modifications of the classical physarum polycephalum mathematical model to adjust it for water distribution system optimization. The methodology was tested on two small-scale benchmark examples: two-loop and Hanoi networks. In the both cases, the obtained results are 10-11% above the known optimal solution, however, the number of iterations required to achieve them are exceptionally small. The proposed approach should be further tested for its applicability to the larger networks. Altogether, the method can serve as a good and easily obtainable first approximation for the least cost water distribution system design.

Keywords: Physarum optimization, water distribution systems, least cost design

1 BACKGROUND

Attention to the problem of least cost design of water distribution systems (WDSs) dates back to 1960s [1] and continues to present days [2]. During last two decade, classical linear, nonlinear, and dynamic programming optimization approaches gave place to heuristics and metaheuristics (see [3] for an example review). Later methods gained popularity due to their plasticity and ability to find close-to-optimal solutions for wide variety of problem formulations and, in particular, in cases where classical optimization methods fail or cannot be applied. This family of methods includes various natural phenomenon- or processes-inspired algorithms that utilize the astonishing characteristic of nature to exploit the environment based not on the sophisticated reasoning but rather following a set of simple rules. Behind heuristic optimization methods lies an idea of decomposition of the problem into two levels: a network simulation level that is responsible for flow, pressure, and cost analysis for a given set of design variables, and an optimization level that is responsible for successive modification of the design variables toward the sought solution [4]. In other words, evolution of the decision variables takes place in a space of possible solutions with simulation engine providing a measure of fitting for each solution. To the contrary, the proposed approach initiates the process of development and evolution of a modeled slime mold *Physarum polycephalum* in the space of WDS itself, translating WDS components to world-creating attributes of a living creature.

Physarum polycephalum was introduced for efficient maze solving [5] and later extended for network formation that was comparable in the obtained results to real Tokyo rail system “in efficiency, fault tolerance, and cost” [6]. Noteworthy, this remarkable outcome was achieved by an organism that lacks a nervous system of any kind; in fact, it is a single-celled amoeboid organism. This inspired researcher to construct a mathematical model that imitates the slime mold behavior [6, 7]. These studies introduced *physarum polycephalum* mathematical model to solve the Steiner tree problem – an important network design NP-hard problem and the minimal exposure problem in wireless sensor networks [7]. The main funding of the study is the confirmed ability of this low complexity algorithm to achieve good performance. Later, the model was improved by introducing an energy term [8] for more realistic representation of the observed biological effects. In comparison to other methodologies for the shortest path problem, improved *physarum polycephalum* algorithm outperforms the previously developed basic model as well as the ant colony optimization algorithm on running time and number of iterations. Compared with an algorithm specifically developed for the shortest path problem (Dijkstra algorithm), the improved *physarum polycephalum* algorithm can find more than one shortest path at the same time.

In this study, we attempt to adjust the algorithm for the least cost design of water distribution systems problem. To authors' knowledge, this is the first attempt to utilize the *physarum polycephalum*-inspired algorithm for WDS optimization. Despite the fact that the achieved solution is higher than known solutions for the benchmark case studies, its value cannot be achieved in random search within a reasonable amount of time. Moreover, the number of iterations required to obtain the solution is very small, which can be a promising sign for further use of the algorithm for an easily obtainable first approximation for the optimal solution.

2 METHODS

A mathematical model for the *physarum polycephalum*-inspired algorithm (hereinafter, slime mold model) is based on the methodology presented in [8]. According to it, the organism is represented as a graph with the flow in its tubular edges approximated by Poiseuille flow

$$Q_{ij} = \frac{D_{ij}}{L_{ij}}\left(p_i - p_j\right) \tag{1}$$

where Q_{ij} is the flux through the edge between the nodes i and j, D_{ij} is the conductivity of the corresponding edge and L_{ij} is its length, and p_i is the pressure at the node i. The inflow and outflow are balanced at each node. The evolution of the organism is represented by the change in the edges' conductivity as a function of the flux through the corresponding edge.

$$\frac{d}{dt}D_{ij} = f\left(\left|Q_{ij}\right|\right) - rD_{ij} \tag{2}$$

where r is a decay rate of the edge tube. Usually, the used function is the absolute value of the flux and $r = 1$, resulting in the following expression for the conductivity change between iterations n and $n + 1$

$$\frac{D_{ij}^{n+1} - D_{ij}^{n}}{\delta t} = \left|Q_{ij}\right| - D_{ij}^{n+1} \tag{3}$$

To adjust the algorithm for the solution of the shortest path problem, the flux function from the Eq. 2 is modified and a new term is introduced that brings more biological rationale into the mathematical model [8]. In this study, the flux function is also changed to link the model to WDS modeling while preserving the model as simple as possible. The resulting conductivity evolution formula is

$$\frac{D_{ij}^{n+1} - D_{ij}^{n}}{\delta t} = h_{ij}\left|\Delta H_{ij}\right| - ac_{ij} - D_{ij}^{n+1} \tag{4}$$

where h_{ij} is the "hunger" coefficient for the edge between the nodes i and j, ΔH_{ij} is the head loss at the corresponding edge and c_{ij} is the cost of this edge, and a is the cost coefficient.
If imagining a slime mold developing and growing in the layout of a WDS, the flow direction should be mentally reversed. A water source can be conceptualized as the creature's center that searches for "food" placed at the network nodes and represented by WDS actual demands. The creature adjusts its tubes to obtain the "food" with the minimal losses. Then, we force an additional dynamic change in the form of rising creature's "hunger" if the pressure at any node in the system is below the predefined minimum.
Altogether, the following adjustments to the basic slime mold model were introduced:
(1) The head loss is used as a flux function. It is calculated using the actual Hazen-Williams head loss formula

$$\Delta H_{ij} = 1.526 \cdot 10^{7} \frac{Q_{ij}^{1.852}}{C_{ij}^{1.852}} d_{ij}^{-4.871} L_{ij} \tag{5}$$

where Q_{ij} is the flux in m^3/h, C_{ij} is the Hazen-Williams coefficient, d_{ij} is the diameter in cm (from the previous iteration), L_{ij} is the length in km [12].

(2) The "hunger" coefficient h_{ij} was introduced to account for the feasibility of a solution. At each iteration step, if the pressure at any nodes adjacent to the current edge is lower than the predefined minimum p_{min}, the hunger coefficient is increased for the whole network and for the current edge by the increments hg and hl correspondingly:

$$h_{ij}^{n+1} = \begin{cases} h_{ij}^{n} & p_k \geq p_{\min} \ \forall k \\ h_{ij}^{n} + hg & p_k < p_{\min} \ k \neq i, j \\ h_{ij}^{n} + hg + hl & p_k < p_{\min} \ k = i \text{ or } j \end{cases} \tag{6}$$

This is a rather crude assumption implying that the pressure lower that the minimum at a certain node in a WDS is directly influenced by an adjacent link whereas in reality, the critical link can be located anywhere in the system. However, with only "global hunger" coefficient uniformly increasing in the whole network, the convergence of the method deteriorated.

(3) To connect the conductivity term D to the diameter d, the adjustment coefficient b was introduced, such that

$$D = b \cdot d \tag{7}$$

since in real WDS the relation d : L is very low and had to be adjusted for use in the Eq. 4. The coefficient remains constant for given network.

(4) To account for pipe pricing, an additional term was introduced in the Eq. 4 in comparison to the Eq. 3. The cost term c_{ij} forces the conductivity to change in inverse proportion to the corresponding edge diameter cost with a weighting coefficient a.

Based on the modifications described above, the main procedures of the slime model for the WDS pipe sizing problem are given in Figure 1.

It can be argued that the presented model solves weighted multiobjective optimization problem with the minimum head loss (responsible for the feasibility of the solution) and minimum cost objectives and h_{ij} and a as dynamic weight coefficients.

// s, t are 1 x n vectors of start and end nodes of each edge
// L is 1 x n vector of edge lengths
// e, q is 1 x m vector of nodal elevations and demands
// D_{av}, *Cost* is 1 x *nd* vector of available diameters and their cost
$\varepsilon \leftarrow 10^{-4}$, $dt \leftarrow 0.1$
$d_0 \leftarrow d_{ij}^0 \; \forall \; i, j = 1, 2, \ldots, n$ //initial edge diameters
$Q_0 \leftarrow \mathrm{Q}_{ij}^0 \; \forall \; i, j = 1, 2, \ldots, n$ //initial edge flux
$p_i \leftarrow 0 \; \forall \; i = k, \ldots, m - k$ //unknown pressures
$p_i \leftarrow p_k \; \forall \; i = 1, 2, \ldots k$ //known pressures
$hg, hl \leftarrow 1 \; \forall \; i = 1, 2, \ldots n$ // initial edge hunger coefficients
$a \leftarrow 0.1$ // cost coefficient
$b \leftarrow 3500$ // conductivity coefficient
$count \leftarrow 1$
repeat
  $count \leftarrow count + 1$
  $\mathrm{Dij} \leftarrow \mathrm{dij} \cdot \mathrm{b}$
  Calculate pressure for every node using
  $$\forall \sum_{j \in M} \left(\frac{D_{ij}}{L_{ij}} \right) \left(p_i - p_j \right) = q_j$$
  Calculate flux Qij for every edge using Eq. 1
  Calculate head loss dHij using Eq. 5
  Calculate "real" pressure at every node
  Prealj = Pi – dHij
  Update hunger coefficients hij using Eq. 6 and Preal for pressure
  Calculate diameters $d_{ij}(count)$ using Eq. 4
  $$d_{ij}^{count} = \left(dt \left(h_{ij} \left| \Delta H_{ij} \right| - a \cdot cost_{ij} \right) + d_{ij}^{count-1} \right) \Big/ \left(1 + dt \right)$$
  Round $d_{ij}(count)$ to the closest diameter from Dav
  Calculate current cost
until $|d_{ij}(count) - d_{ij}(count\text{-}1)| > \varepsilon \; \forall \; i, j = 1, 2, \ldots, n$

Figure 1. Slime mold algorithm for WDS least cost design

3 IMPLEMENTATION OF THE SLIME MOLD ALGORITHM

The developed algorithm was implemented for two small-scale benchmark WDSs: the two-loop and the Hanoi networks. Due to space limitation, detailed system descriptions are omitted. Layout, system parameters, and the reference to the history of optimal solutions of the two-loop and the Hanoi networks can be found, for example in [10].

There are two approaches in literature for pipe diameter selection. In one, a diameter should be selected for each pipe, and in the second one, a diameter should be selected for a segment of an unknown length of each pipe. The first approach is used for this study, and the corresponding known best solutions are chosen for the comparison in the most cases.

3.1 Two-loop network results

The resulting diameters and system cost obtained for the two-loop network in comparison to known solutions and random search are given in Table 1. The slime mold algorithm results in the final cost 9.7% higher than the known best solution. However, this result is achieved within only 9 to 120 iteration (depending on starting point). This final cost cannot be found in a random search in over a million iteration.

Table 1. Solutions for the two-loop network

Pipe	Diameter, inch			
	Alperovits and Shamir [9]	Zhou et al. [10]	Random 1,000,000 iterations	Slime mold model
1	20, 18	18	18	18
2	8, 6	10	14	12
3	18	16	14	16
4	8, 6	4	2	10
5	16	16	16	14
6	12, 10	10	2	6
7	6	10	14	12
8	6, 4	1	14	10
Cost ($)	479,525	419,000	470,000	460,000

The method convergence from the known "good" starting point (the starting point of the Alperovich and Shamir method [9]) is given in Figure 2 for links 1, 2, 3, and 8. From a random starting point, it takes the algorithm 75 iterations on average to converge. In 77 runs out of 100, the final solution is the 460,000 or slightly worse solution with a 14-inch diameter of Pipe 2 instead of a 12-inch. For some starting points, a 2-stage solution can benefit to the algorithm convergence. It takes only 6 iterations to converge from an intermediate solution to the best slime mold algorithm solution.

The values of the coefficients used for this network are the following: b = 3500, a = 0.1, hg = 0.8, hl = 4.2 and the "hunger" coefficients rise during the algorithm's run from 1 to 6.6, 10.8 or 15 for different edges of the network.

Final pressures for all system nodes for the compared solutions are given in Table 2. The slime mold model meets the minimum pressure constraint at Node 7. However, it fails to discover better solution by decreasing the size of Pipe 8. It is worth noting that the final solution of the slime mold model results in the sum head loss in the network equal to 30.85 m/km whereas the known best solution by [10] results in the sum head loss 60.19 m/km by this confirming that head loss serves as an essential part of the objective function.

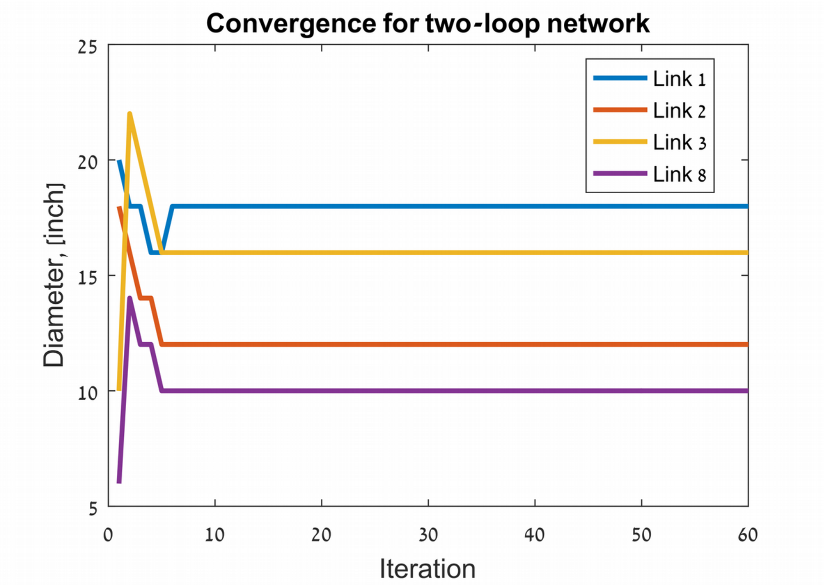

Figure 2. Slime mold algorithm convergence, two-loop network

3.2 Model sensitivity

The proposed algorithm was checked for its sensitivity to various model parameters. Coefficient b can take values within a quite wide range without any effect on the algorithm's convergence or its rate. For this example, $b = 3500$ was chosen for the two-loop network, and for b in [3405, 3950], the algorithm converges to the indicated solution within 9 iterations from the good starting point. Coefficients *hg* and *hl* seems to be interdepended: the algorithm converges with very similar rate (9 to 12 iterations) for (*hg*, *hl*) pairs (0.3, 1.5), (0.7, 3.5), (1, 5), and similar. However, in cases different from the finally chosen pair (0.8, 4.2), the algorithm mostly converges to the slightly worse solution with the final cost $467,000.

Table 2. Pressure heads for the two-loop network

Node	Pressure, m			
	Alperovits and Shamir [9]	Zhou et al. [10]	Random 1,000,000 iterations	Slime mold model
2	53.96	53.24	53.23	53.23
3	32.32	30.49	36.84	37.67
4	44.97	43.44	42.38	44.70
5	32.31	33.78	43.89	43.55
6	31.19	30.43	31.47	31.68
7	31.57	30.54	32.62	30. 53

Cost coefficient a in the interval [0.05, 0.14] leads to the algorithm convergence to the indicated solution. For this coefficient in the intervals [0, 0.04] and [0.15, 0.33], the algorithm converges to a solution with the final cost $467,000 and for larger a, the cost part of the function in Eq. 4 prevails leading to an infeasible solution.

The sensitivity to the initial point is discussed in Section 3.1. Since the slime mold model does not converge to the known best solution for a given network, the algorithm's ability to result in several feasible suboptimal solutions of similarly good quality can be seen as an advantage for providing several options for further consideration.

3.3 Hanoi network results

Comparison to known solutions and random search for the Hanoi network are given in Table 3. The solution obtained by the slime mold model is 11.5% higher than the known best solution; however, it requires only 35 iterations on average and converges to the indicated solution in 93 cases out of 100. In comparison with random search, no feasible solution was found in 1 million iterations. For this network, the following values of the model coefficients were used: $a = 0.1$, $hg = 1$, $gl = 5$, $b = 8200$. Evidently, only the conductivity-diameter adjustment coefficient b required alteration. In fact, the pair (0.8 4.2) for the coefficients (hg, gl) results in even better solution -6.624 million dollars, which is 9.4% higher than the known best solution, but is periodic for Pipe 6 and was disregarded on this ground.

Table 3. Solutions for the Hanoi network

	Savic and Walters [11]	Perelman et al. [12]	Zhou et al. [10]	Random iterations	Slime mold model
Number of iterations	1,000,000	2,500	300-600	1,000,000	35
Cost, $ millions	6.073	6.055	6.056	No feasible solution	6.752

The algorithm's convergence from a random initial point is given in Figure 3. Altogether, the slime mold algorithm shows better performance when implemented for the Hanoi network than the two-loop network. This can be connected to significantly lower available pipe diameter options (6 vs. 14) in the Hanoi example.

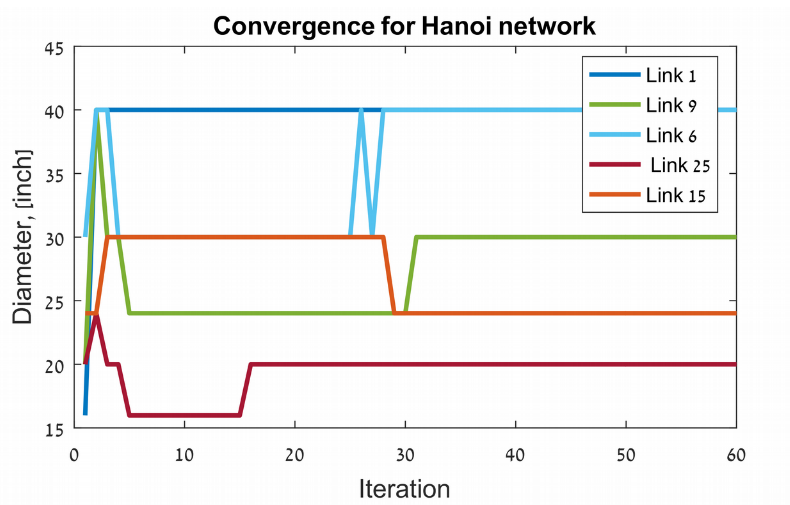

Figure 3. Slime mold algorithm convergence, Hanoi network

4 CONCLUSIONS

This paper presents an attempt to adapt the *physarum polycephalum*-inspired algorithm for WDS least cost design. Modifications of the known slime mold mathematical model construct an evolution function that comprises two parts: the head loss part that ensures solution feasibility and the cost part that ensures total design cost minimization. The algorithm results in solutions of an order of 10% higher than the known best solution; however, it requires a mere number of iterations to converge. The algorithm was tested on two small-scale benchmark networks. Notably, the algorithms' performance was better for the larger network. Another feature of the algorithm is its ability to return several feasible solutions of similar quality with a change in the model parameters. The slime mold model applicability to WDS optimization should be further investigated, in particular, in its applicability and performance for large-scale networks and practical worthwhileness of the proposed approach.

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