A THEORY FOR REALISTIC MODELLING OF LEAKAGE AND INTRUSION FLOWS THROUGH LEAK OPENINGS IN PIPES

Van Zyl JE[1], Lambert A[2], Collins R[3]

[1] Department of Civil Engineering, University of Cape Town, South Africa
[2] Water Loss Research & Analysis Ltd, United Kingdom, LL30 1SL.
[3] Department of Civil and Structural Engineering, University of Sheffield, UK
[1]*kobus.vanzyl@uct.ac.za*

ABSTRACT

The hydraulics of leakage and intrusion flows through leak openings in pipes is complicated by variations in the leak areas due to changes in pressure. This study presents a theoretical framework for the analysis of both leakage and intrusion flows through real leaks in pipes. A linear area-pressure relationship is assumed. The leakage exponent of a given leak opening is shown to generally not be constant with variations in pressure, and to approach infinity when the leakage number approaches a value of minus one. Significant modelling errors may result if an exponent used in the power equation used beyond its calibration pressure range, or at high exponent values.

Keywords: Leakage modelling, FAVAD, N1

1 INTRODUCTION

Pipes in water distribution systems invariably develop leaks, which have negative consequences including leakage under positive pressure conditions and intrusion of contaminated ground water under negative pressure conditions.

Leakage and intrusion modelling is complicated by a number of factors, including leak area variation with pressure, the flow regime, soil hydraulics and variations in leakage elevations in a DMA (district metered area) [1, 2]. This resulted in two approaches to model the behaviour of leakage in water distribution systems: the FAVAD (Fixed and Variable Area Discharges) and power equation models.

The purpose of this paper is to present a theory for the realistic modelling of leakage and intrusion through leak openings in pipes in light of the latest research findings on the behaviour of leak areas with pressure. A more detailed report on this study was recently published as an open access paper in the Journal of Hydraulic Engineering [3] and may be downloaded from http://bit.ly/2sox9fX.

The study assumed a pipe that is filled with and submerged in the same fluid, and thus factors such variations in the discharge coefficient, the flow regime and soils outside the pipe were ignored.

The next section provides an overview of the two leakage modelling approaches and how they are related. A discussion is then provided on the behaviour of leak openings with positive head-area slopes under leakage and intrusion flow conditions.

2 BACKGROUND

Hydraulically a leak opening acts as an orifice for which the hydraulic behaviour is well understood (for instance, see [4]) Defining the head differential h over the leak opening as $h = h_{internal} - h_{external}$ and assuming the pipe is both filled with and submerged in water (thus ignoring the impact of soil hydraulics) allows the orifice equation for both leakage and intrusion flows to be written as:

$$Q_o = sgn(h)C_d A\sqrt{2g|h|} \quad \text{...(1)}$$

The main problem with the orifice equation in modelling leakage and intrusion flows is that these openings are often not fixed, but vary with pressure. It has been established through several studies that the area of leak openings is a linear function of pressure in most cases [1, 5 – 14]. Thus the leak area can be described using the function:

$$A = A_0 + mh \quad \text{...(2)}$$

Where A_0 is the initial area (the area of the leak opening when the head differential is zero) and m the head-area slope. Replacing (2) into (1) results in the FAVAD equation [5], called the modified orifice equation in this paper:

$$Q_{mo} = sgn(h)C_d\sqrt{2g}(A_0|h|^{0.5} + m|h|^{1.5}) \quad \text{...(3)}$$

The first term in the equation describes orifice flow based on the initial leak area and the second term the flow through the expanded portion of the leak opening. The ratio between these two terms is called the leakage number L_N [15], defined as:

$$L_N = \frac{mh}{A_0} \quad \text{...(4)}$$

The behaviour of leaks is commonly modelled using a power equation, which is called the N1 power equation in leakage practice, and emitter function in hydraulic network modelling [2, 13, 16 – 22]. The power equation can be written as:

$$Q_p = sgn(h)C|h|^{\alpha} \quad \text{...(6)}$$

Where C is the leakage coefficient and α the leakage exponent.

Mathematically, Equation 6 is a generalized form of the orifice equation. However, this generalization removes the fundamental theoretical base on which the orifice equation built. Thus the power equation is nothing more than an empirical equation, and should be treated as such.

In addition, the parameters C and α are not constant but vary with pressure and the equation is dimensionally awkward [15]. Finally, it is shown in this paper that there are instances of modified orifice behaviour where the leakage exponent approaches infinity.

Van Zyl et al [3] showed that the modified orifice and power equations may be linked through the leakage number using the following relationship:

$$\alpha = \frac{1.5L_N+0.5}{L_N+1} \quad \text{...(7)}$$

Equation 7 allows the leakage exponent to be calculated from the modified orifice parameters, and vice versa. This has several benefits, such as calculating variations in the leakage exponent with changes in pressure and investigating the cause of very high leakage exponents.

3 LEAK OPENINGS WITH POSITIVE HEAD-AREA SLOPES

The head-area slope of a leak opening may be positive (the leak area increases with increasing pressure), or negative (the leak area decreases with increasing pressure). While negative head-area slopes are uncommon, they have been observed especially in circumferential cracks.

For positive head-area slopes, four possible cases may be identified as shown in Figure 1. The figure shows the variation of the leak area for both positive and negative pressures. For negative pressures, an absolute limit exists where the pressure in the pipe reaches a vacuum. The four cases may be described as follows:

- ***Case P0***: An initial area greater than zero with the leak opening not closing fully even when the pressure in the pipe reduces to absolute zero.
- ***Case P1***: An initial area greater than zero with the leak opening closing fully before the pressure in the pipe reduces to absolute zero.
- ***Case P2***: An initial area of zero, for instance a crack that closes fully when there is no pressure differential over the pipe wall.
- ***Case P3***: An initial area less than zero, resulting in a negative initial area in the equation. While a negative initial area is not physically possible, it may occur theoretically when a leak opening remains closed at head differences above zero, for instance as a result of external forces.

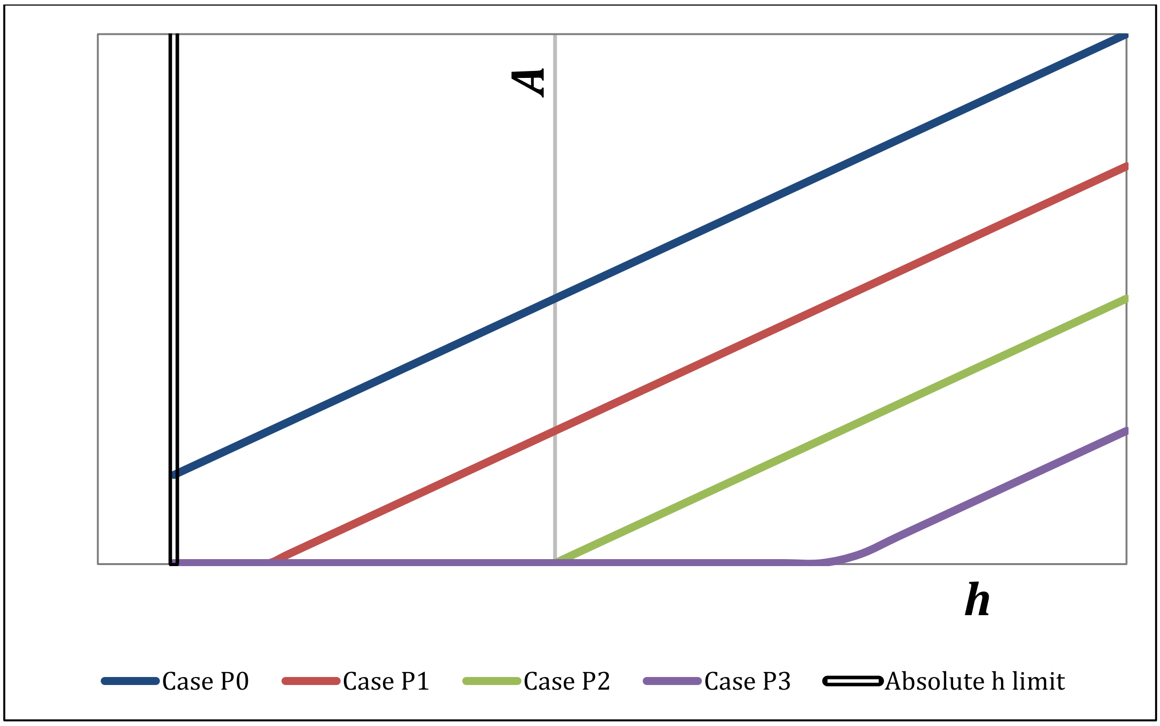

Figure 1. Leak area as a function of pressure for leak openings with a positive head-area slope

The flow rate may be calculated for each of the respective cases using Equation (1) as shown in Figure 2. It may be observed that the flow rate through the leak opening becomes zero when it closes fully at a negative pressure (Case P1), zero pressure (Case P2) or a positive pressure (Case P3).

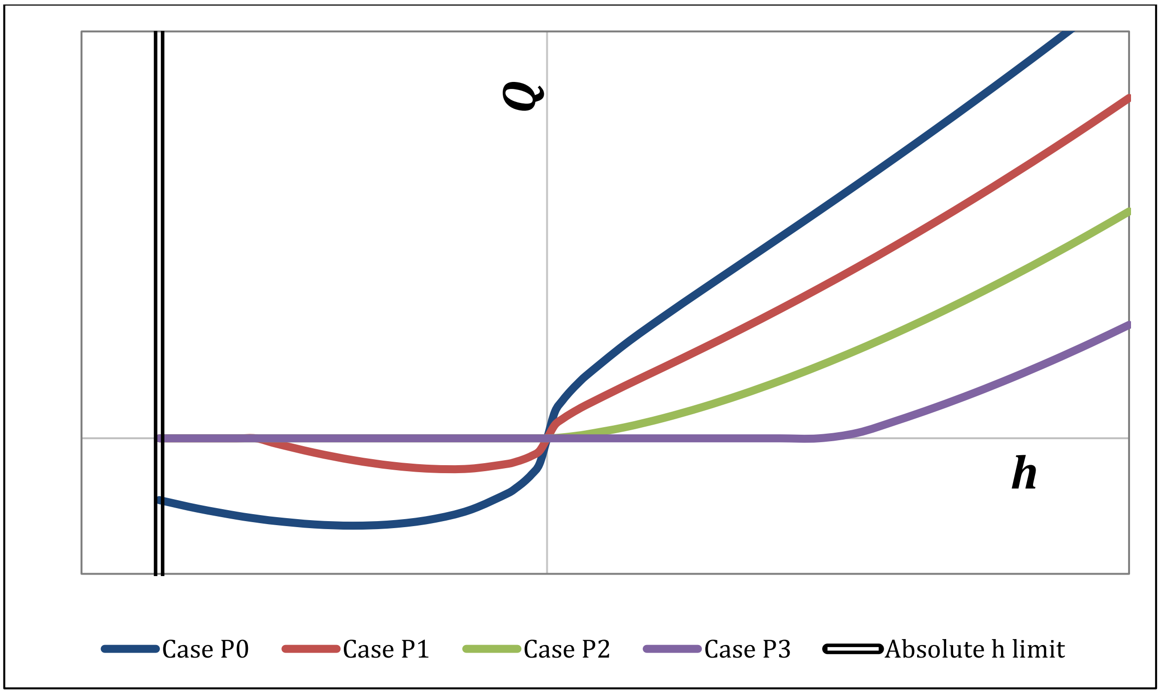

Figure 2. Flow rate as a function of pressure for leak openings with a positive head-area slope

It is also possible to calculate the leakage number for each case as a function of pressure using Equation (4), as shown in Figure 3. Since both the initial area and head-area slope are constant for a particular leak, the leakage number is only a function of pressure. The leakage number doesn't have any meaning when the leak area is zero, and thus Figure 3 only shows the leakage numbers when flow occurs through the leak opening. Case P2 is not shown in Figure 3 since an initial area of zero means the leakage number is at infinity.

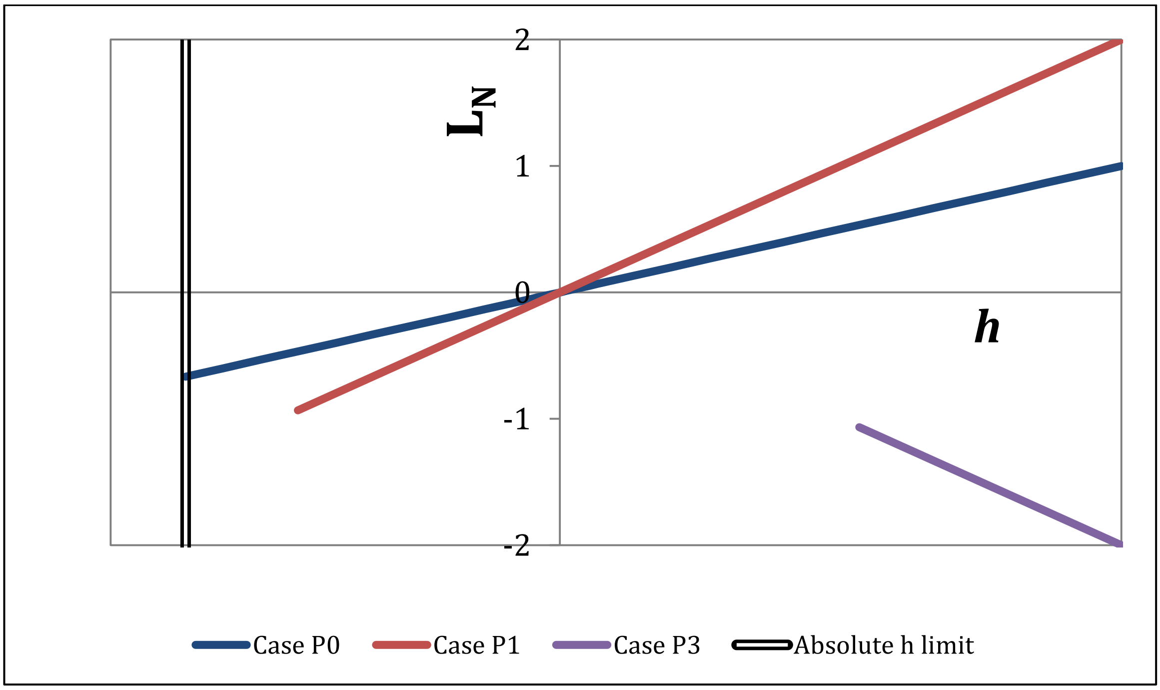

Figure 1. Leakage number as a function of pressure for leak openings with a positive head-area slope

It is now possible to calculate the leakage exponent that corresponds to each case as a function of pressure using Equation (7), and these are shown in Figure 4. The following observations may be made from the figure:

- Cases P0 and P1: Under positive pressure conditions the leakage exponent increases from 0.5 at a pressure of zero towards an asymptote at a leakage exponent of 1.5. For negative pressures the leakage exponent reduces and approaches minus infinity as the leakage number approaches -1, which occurs when the leakage area approaches zero.
- For Case P2, the leakage number is at infinity, and thus the leakage exponent is 1.5 for all positive pressures.
- For Case P3, the leakage exponent is greater than 1.5 for all pressures and approaches infinity as the pressure reduces to the point where the leakage number approaches -1, which is where the leakage area approaches zero.

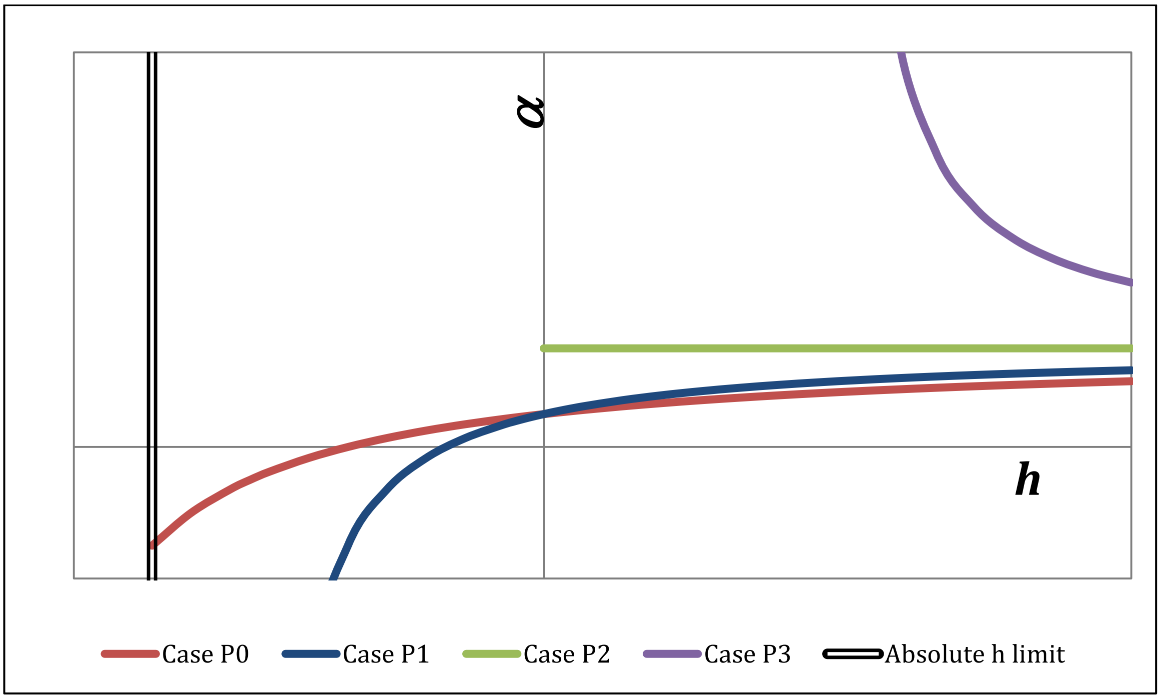

Figure 1. Head-area slope as a function of pressure for leak openings with a positive head-area slope

The results show that the leakage exponent can vary substantially for the same leak opening. Thus, while the power leakage equation may be used as an empirical equation, significant modelling errors may result if the power equation is extrapolated beyond its calibration pressure range.

4 CONCLUSIONS

This paper developed a theory of leakage and intrusion flows through leak openings based on the orifice equation, modified to incorporate a linear pressure-area relationship. The dimensionless leakage number is described, as well as a mathematical relationship between the leakage number and the leakage exponent.

For leaks with a positive head-area slope, four possible cases are defined with their respective flow rates, leakage numbers and leakage exponents as a function of pressure. These relationships show that for a leak with an initial area of zero, the leakage exponent will always be equal to 1.5 and for other cases, the leakage exponent approaches infinity as the leakage number approaches -1, which occurs when the leakage area approaches zero.

Acknowledgements

The authors would like to acknowledge Mr John May, whose 1994 paper created the impulse that led to the work presented in this paper.

References

[1] Van Zyl, J.E., Clayton, C.R.I. (2007). “The effect of pressure on leakage in water distribution systems”, *Water Management*, **160** (WM2) 109-114.

[2] Schwaller, J., Van Zyl, J.E. (2014) Modeling the pressure-leakage response of water distribution systems based on individual leak behavior. *Journal of Hydraulic Engineering*, 04014089.

[3] Van Zyl, J. E., Lambert, A., and Collins, R. (2017) Realistic modeling of leakage and intrusion flows through leak openings in pipes. *Journal of Hydraulic Engineering*, **143**(9), DOI: 10.1061/(ASCE)HY.1943-7900.0001346

[4] Idelchik, I.E. (1994) Handbook of Hydraulic Resistance, 3rd Edn. Begell House, 790 pp.

[5] May J. H. (1994) Leakage, Pressure and Control. In: Proceedings of the BICS International Conference Leakage Control Investing In Underground Assets, The SAS Portman Hotel.

[6] Cassa, A.M., Van Zyl, J.E., Laubscher, R.F. (2010) A numerical investigation into the effect of pressure on holes and cracks in water supply pipes, Urban Water Journal, 7 (2) 109 - 120.

[7] Ferrante, M., Massari, C., Brunone, B. & Meniconi, S., 2011. Experimental Evidence of Hysteresis in the Head-Discharge Relationship for a Leak in a Polyethylene Pipe. *Journal of Hydraulic Engineering*, Issue 137-7, pp. 775-780.

[8] Ferrante, M., Massari, C., Brunone, B. & Meniconi, S., 2011. Experimental Evidence of Hysteresis in the Head-Discharge Relationship for a Leak in a Polyethylene Pipe. *Journal of Hydraulic Engineering*, Issue 137-7, pp. 775-780.

[9] De Marchis, M., Fontanazza, C.M., Freni, G., Notaro, V., Puleo, V. (2016) Experimental Evidence of Leaks in Elastic Pipes, Water Resources Management 30:2005–2019

[10] Fox, S., Collins, R., Boxall, J. (2016a) Experimental study exploring the interaction of structural and leakage dynamics, Journal of Hydraulic Engineering, 04016080.

[11] Fox, S., Collins, R., Boxall, J. (2016b) Physical investigation into the significance of ground conditions on dynamic leakage behaviour, Journal of Water Supply: Research and Technology—AQUA **65** (2) 103 – 115.

[12] Meniconi, S., Brunone, B., Ferrante, M., Berni, A. and Massari, C. (2011) Experimental evidence of backflow phenomenon in a pressurised pipe. In Proc. 11th Intl. Conference on Computing and Control for the Water Industry—CCWI.

[13] Mora-Rodríguez, J., Delgado-Galván, X., Ramos, H.M., López-Jiménez, P.A. (2014) An overview of leaks and intrusion for different pipe materials and failures, Urban Water Journal, 11:1, 1-10, DOI: 10.1080/1573062X.2012.739630

[14] Mora-Rodríguez, J., Delgado-Galván, X., Ortiz-Medel, J., Ramos, H.M., Fuertes-Miquel, V.S., López-Jiménez, P.A. (2015) Pathogen intrusion flows in water distribution systems: according to orifice equations, Journal of Water Supply: Research and Technology—AQUA **64(**8).

[15] Van Zyl, J.E., Cassa, A.M. (2014) "Modeling elastically deforming leaks in water distribution pipes", *Journal of Hydraulic Engineering*, **140** (2) 182 – 189.

[16] Giustolisi, O., Savic, D.A., Kapelan, Z. (2008) Pressure-Driven Demand and Leakage Simulation for Water Distribution Networks, Journal of Hydraulic Engineering, **134** (5) 626 – 635.

[17] Gupta, R., Nair, A.G.R, Ormsbee. L. (2016) Leakage as Pressure-Driven Demand in Design of Water Distribution Networks, Journal of Water Resources Planning and Management, 04016005, 1 – 14.

[18] Ferrante, M., Meniconi, S., and Brunone, B. (2014) Local and global leak laws. The relationship between pressure and leakage for a single leak and for a district with leaks. Water Resources Management, 28(11), pp. 3761-3782.

[19] Ogura, 1979. Experiment on the Relationship between Leakage and Pressure. *Japan Water Works Association*, June, pp. 38-45.

[20] Hiki, S., 1981. Relationship between Leakage and Pressure. *Japan Waterworks Association Journal*, May, pp. 50-54.

[21] Lambert A. 2000 What Do We Know About Pressure: Leakage Relationship in Distribution System? In: *System Approach to Leakage Control and Water Distribution Systems Management*, Brno, Czech Republic.
[22] Farley, M. & Trow, S., 2003. *Losses in Water Distribution Network.* London: IWA Publishing.